EE 230
Lecture 32

Small Signal Operation of Nonlinear Networks

Quiz 32

Which region of operation of the BJT corresponds to the Saturation region of the MOSFET?

And the number is ?

1 3 8

5 4

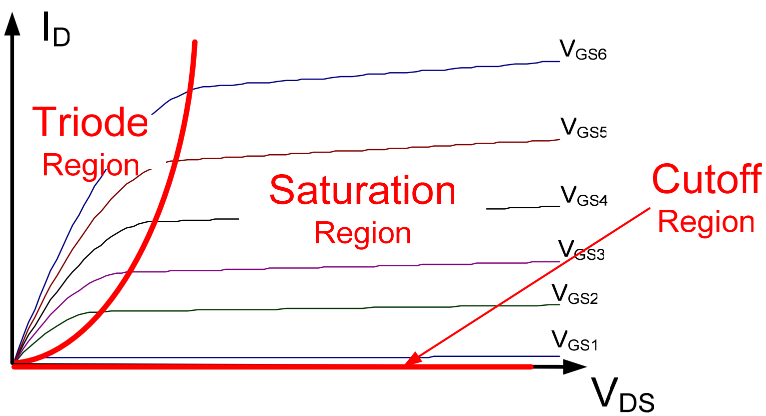
?

2 6

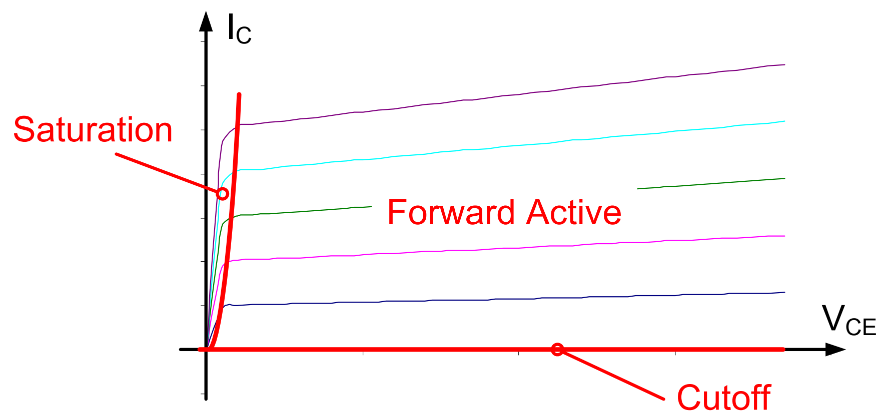
9 7

Quiz 32

Which region of operation of the BJT corresponds to the Saturation region of the MOSFET?



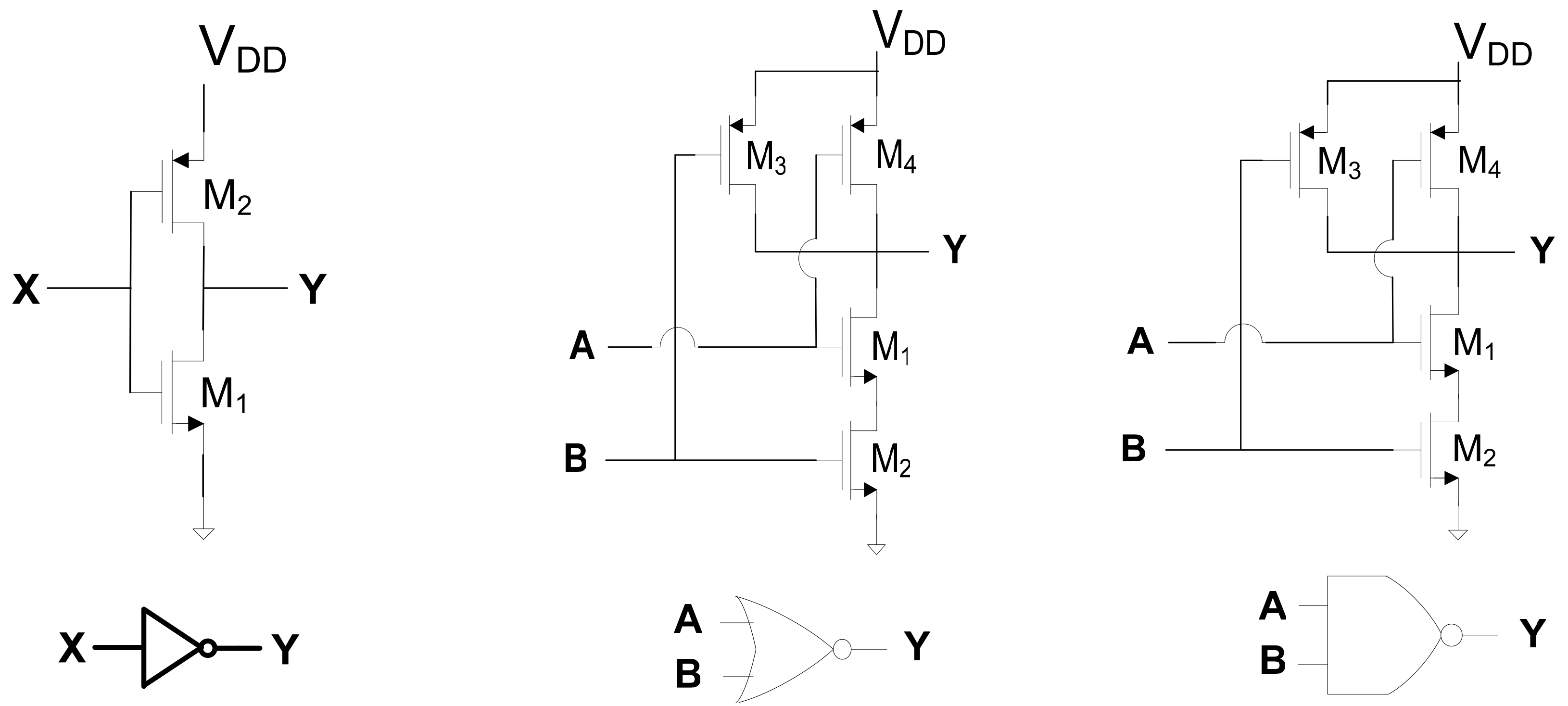
MOSFET



BJT

Review from Last Time:

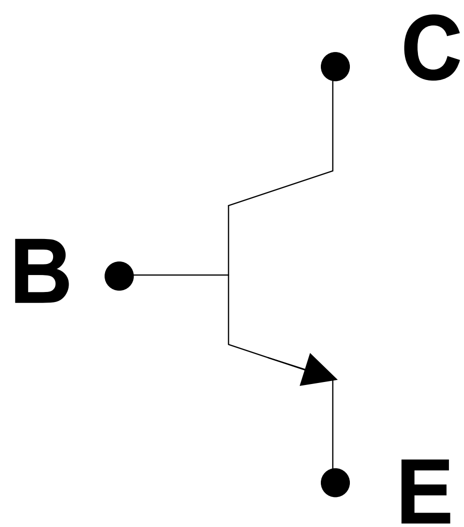
MOS Transistor Applications (Digital Circuits)



- Termed CMOS Logic
- Widely used in industry today (millions of transistors in many ICs using this logic
- Almost never used as discrete devices

Review from Last Time:

Bipolar Transistor

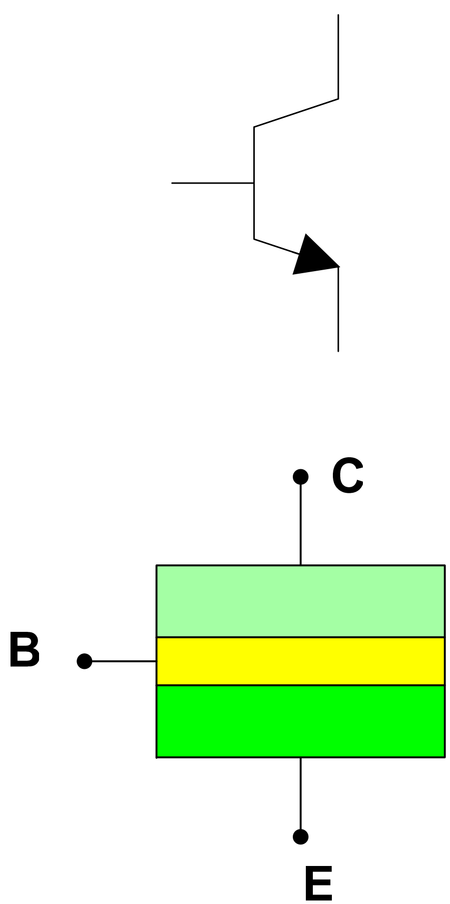


B: Base
C: Collector
E: Emitter

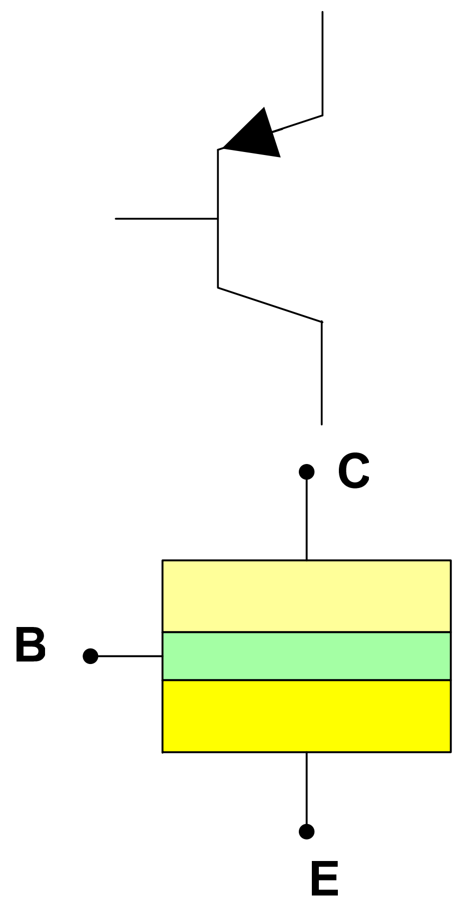
Review from Last Time:

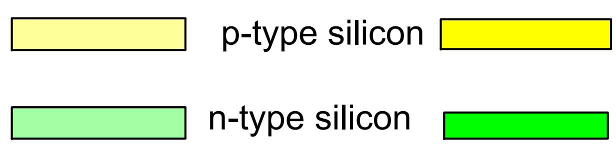
Bipolar Transistor

npn



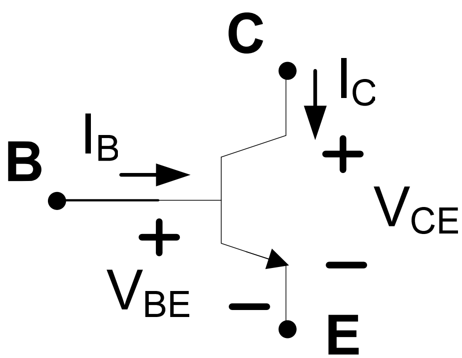
pnp



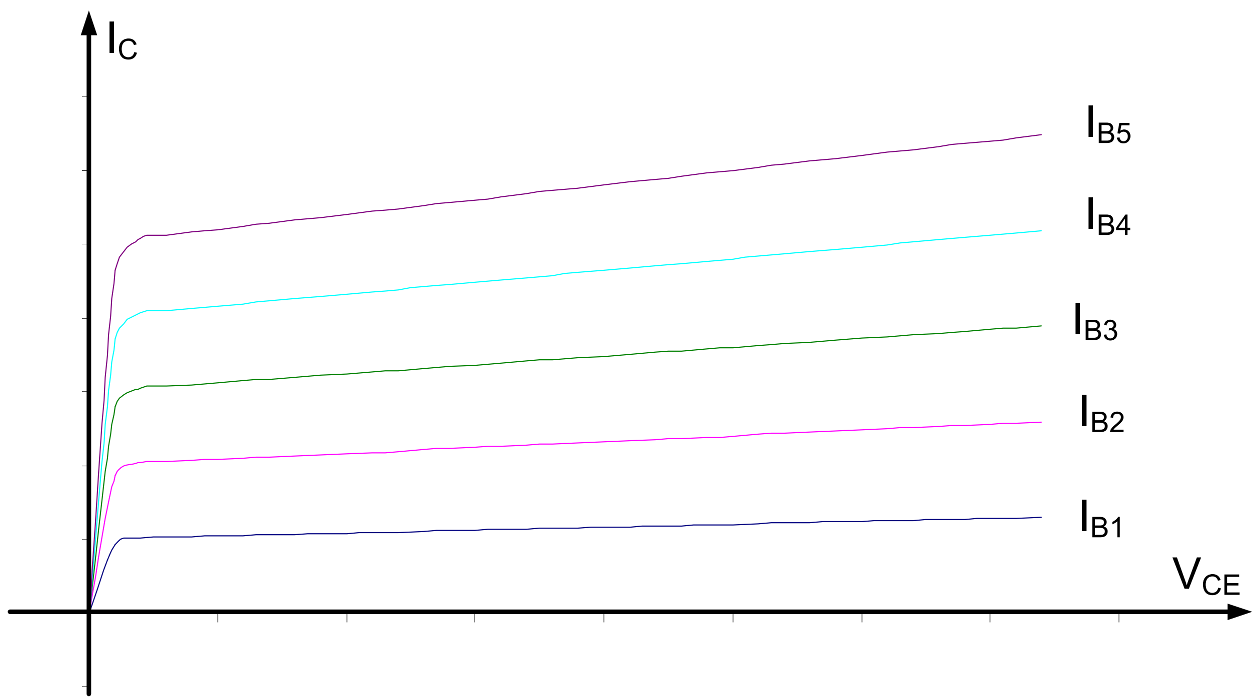


Review from Last Time:

Bipolar Transistor

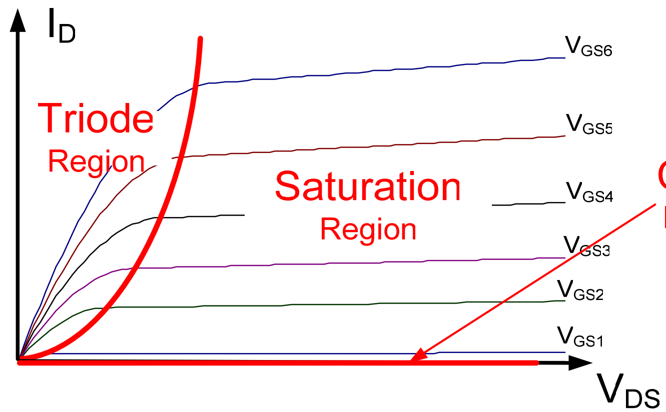


npn

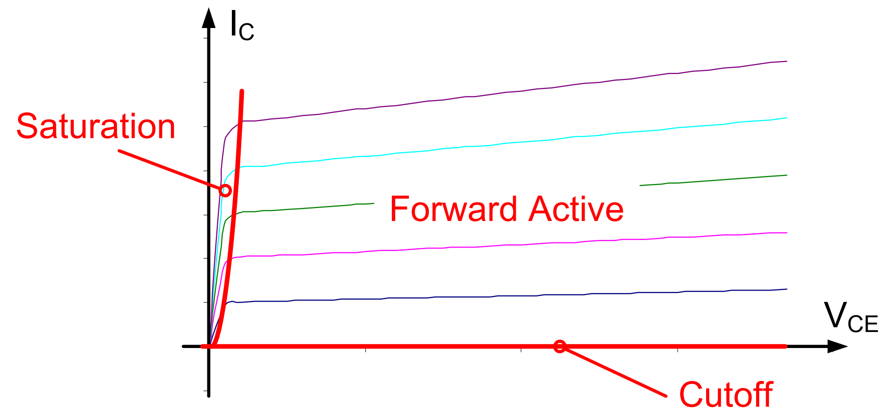


Review from Last Time:

Bipolar and MOS Region Comparisons



MOSFET



BJT

Cutoff	Cutoff
Saturation	Forward Active
Triode	Saturation

Review from Last Time:

Bipolar Transistor

Simplifier Basic Multi-Region Model

$I_C = \beta I_B$ $I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$ $V_t = \frac{kT}{q}$	$V_{BE}>0.4V$ $V_{BC}<0$	Forward Active
$V_{BE}=0.7V$ $V_{CE}=0.2V$	$I_C<\beta I_B$	Saturation
$I_C=I_B=0$	$V_{BE}<0$ $V_{BC}<0$	Cutoff

Methods of Analysis of Nonlinear Circuits

Will consider three different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

   interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course

Circuits with small-signal inputs that vary around some operating point

- *This is one of the most useful classes of circuits that exist*
- *Use is driven by goal to use nonlinear devices (at fundamental device level, that's all we have that provide power gain) to perform linear signal processing functions*
- *Concept of "systems" with small-signal inputs that vary around some operating point throughout the electrical engineering field and in many other fields as well*
- *Although the concepts will be introduced in the context of electronic circuits, the principles and mathematics are generally applied*

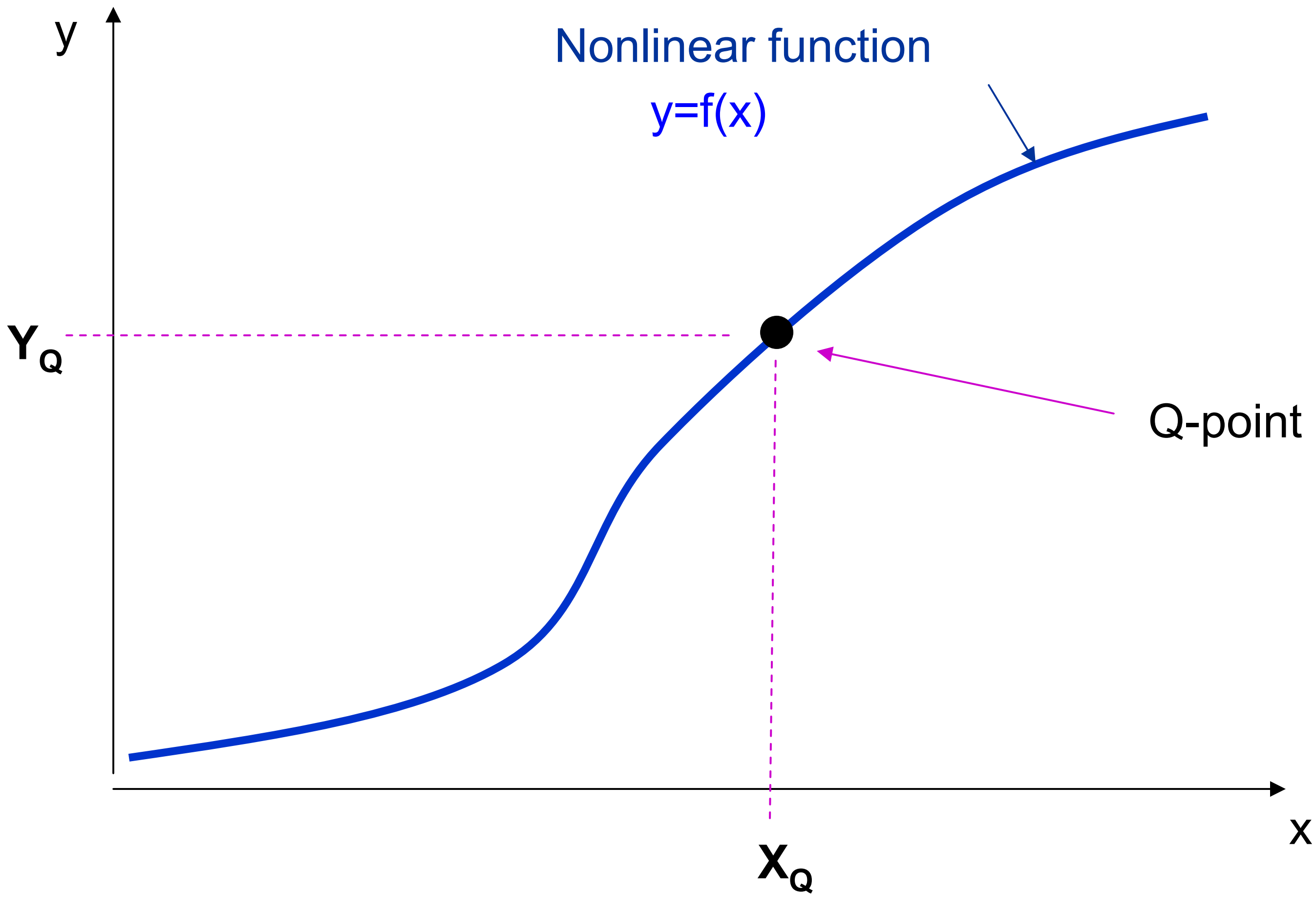
Small-signal Operation of Nonlinear Circuits

- Small-signal principles
- Example Circuit
- Small-Signal Models
- Small-Signal Analysis of Nonlinear Circuits

Small-signal Operation of Nonlinear Circuits

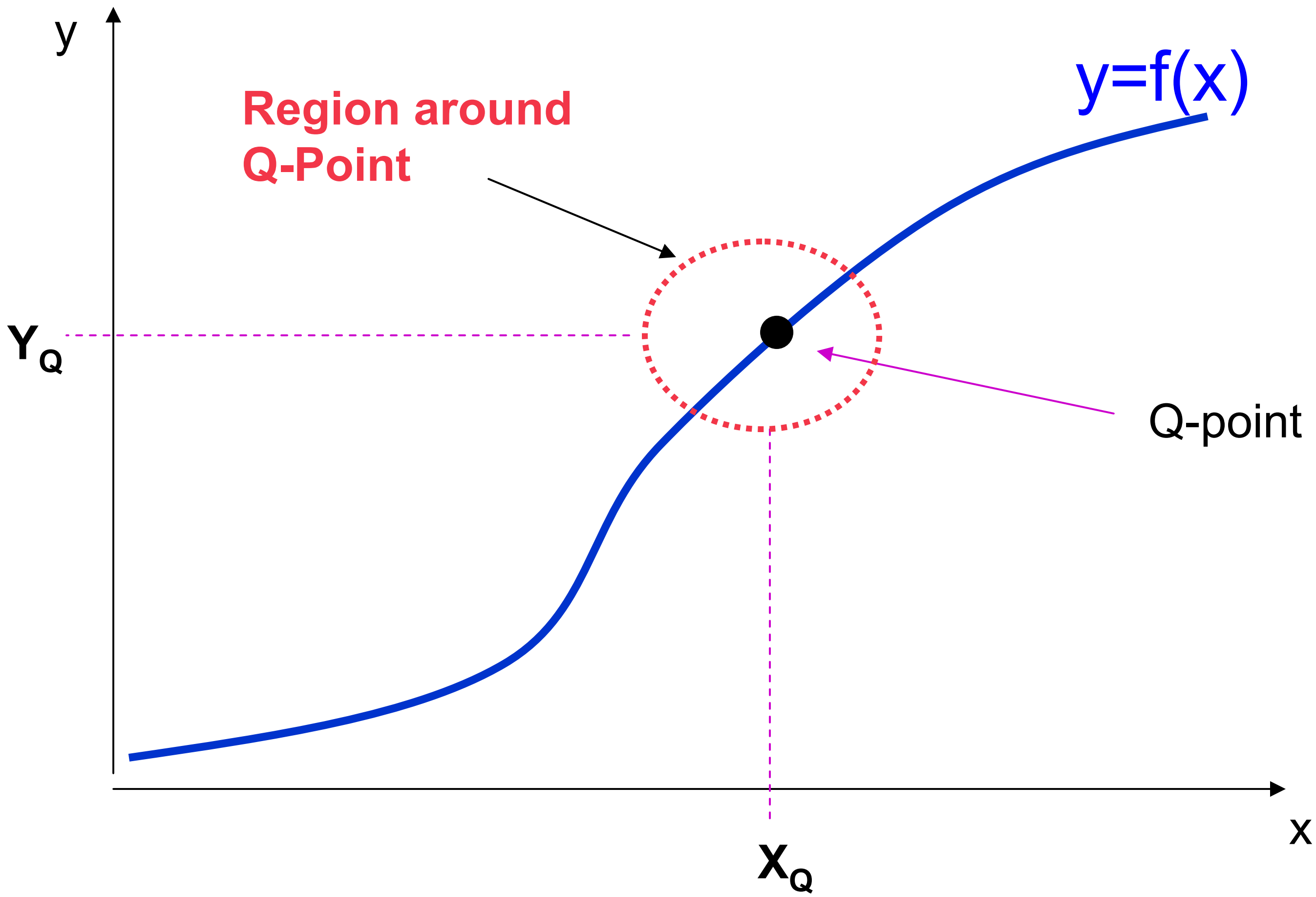
- Small-signal principles
- Example Circuit
- Small-Signal Models
- Small-Signal Analysis of Nonlinear Circuits

Small-Signal Principle

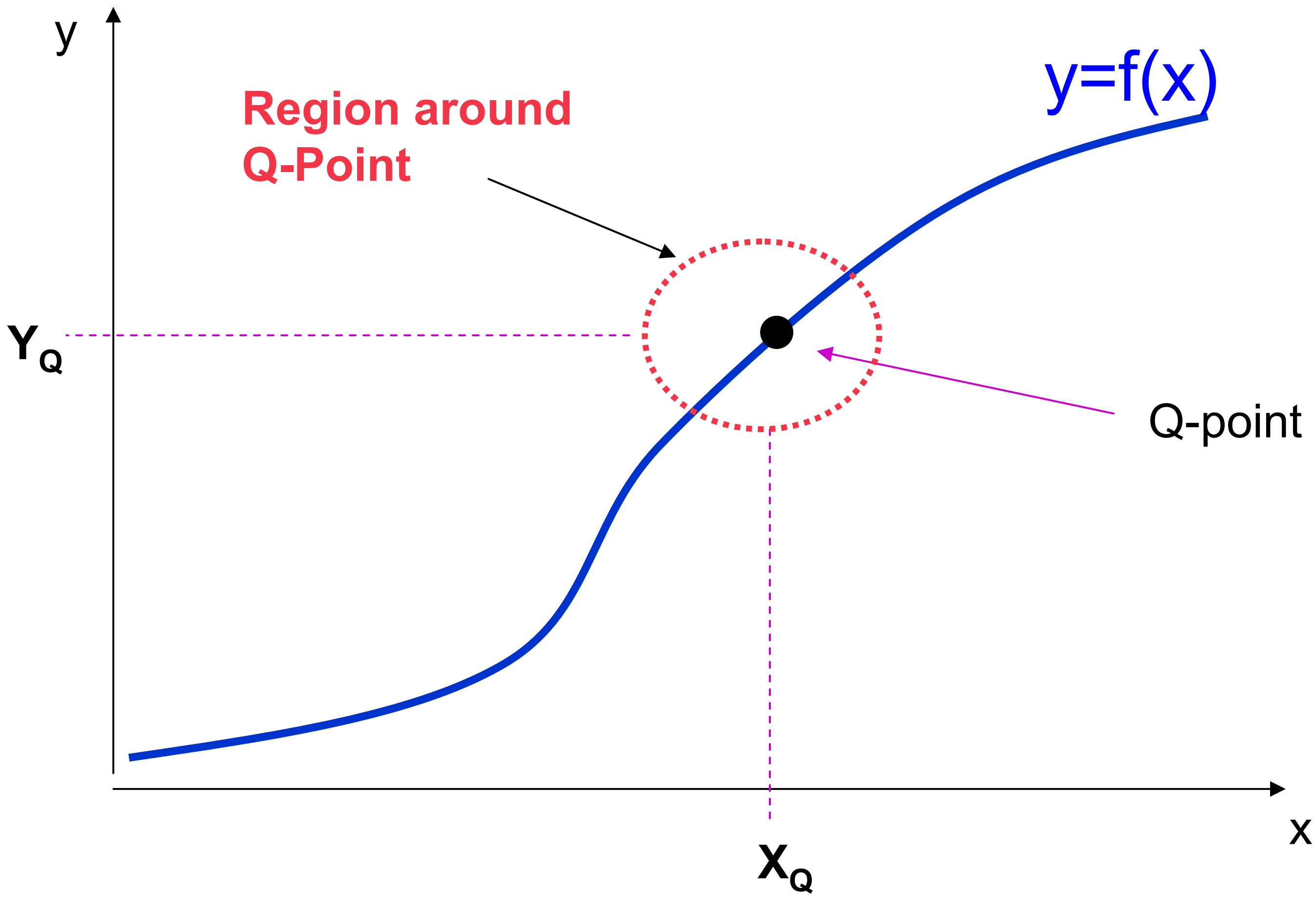


- *units for x,y can be anything*
- *formulation useful in a broad range of fields !*

Small-Signal Principle

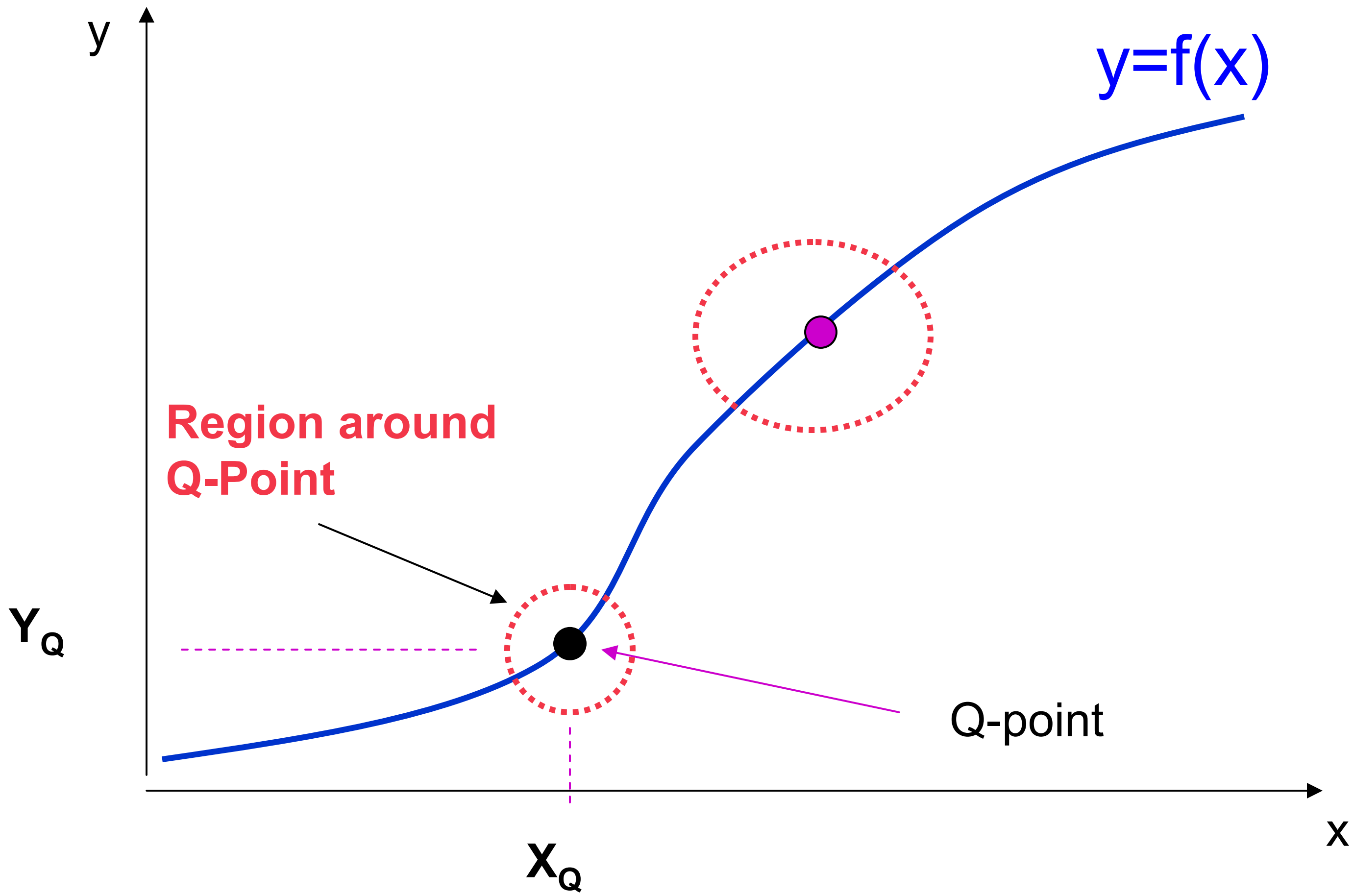


Small-Signal Principle



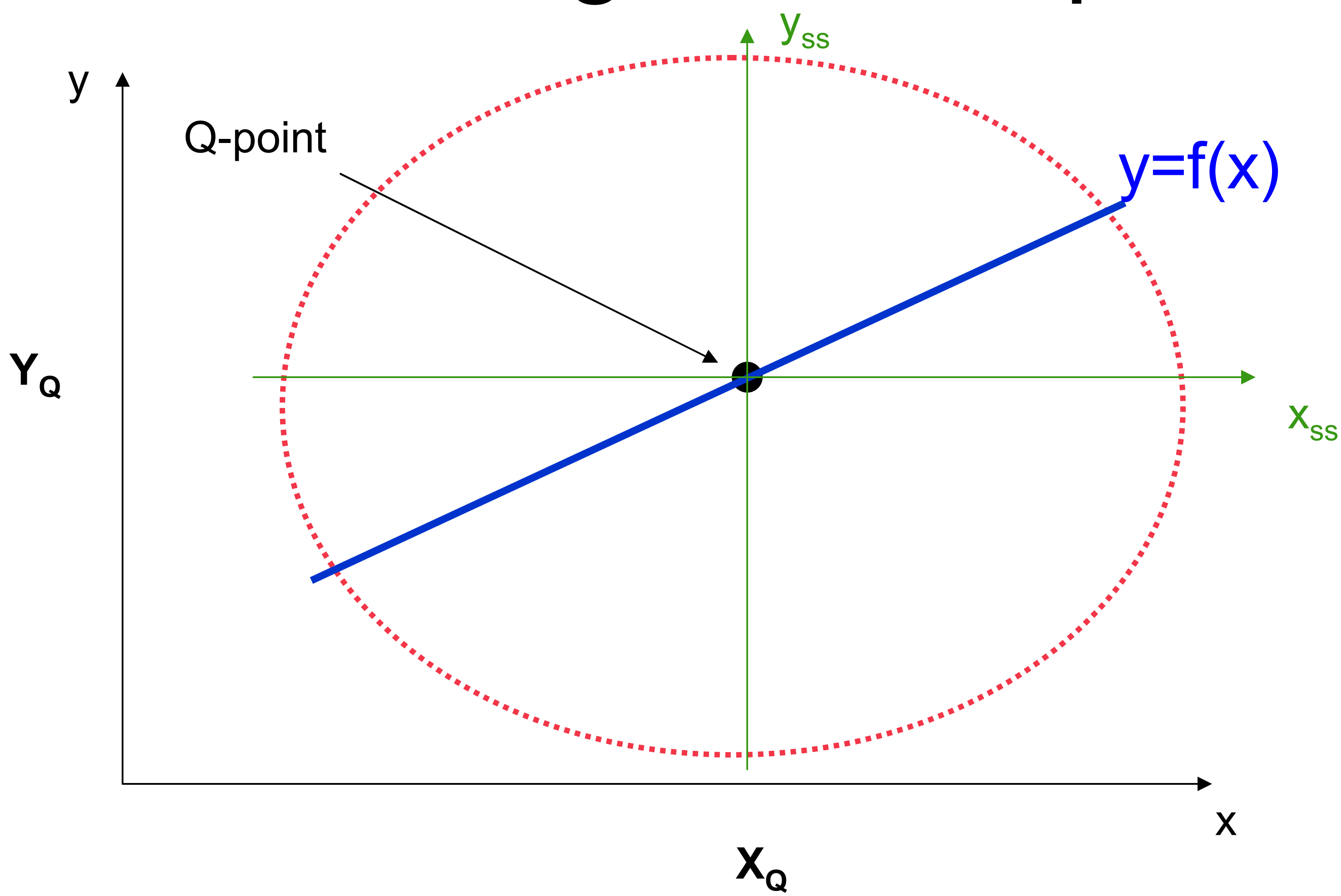
Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points

Small-Signal Principle



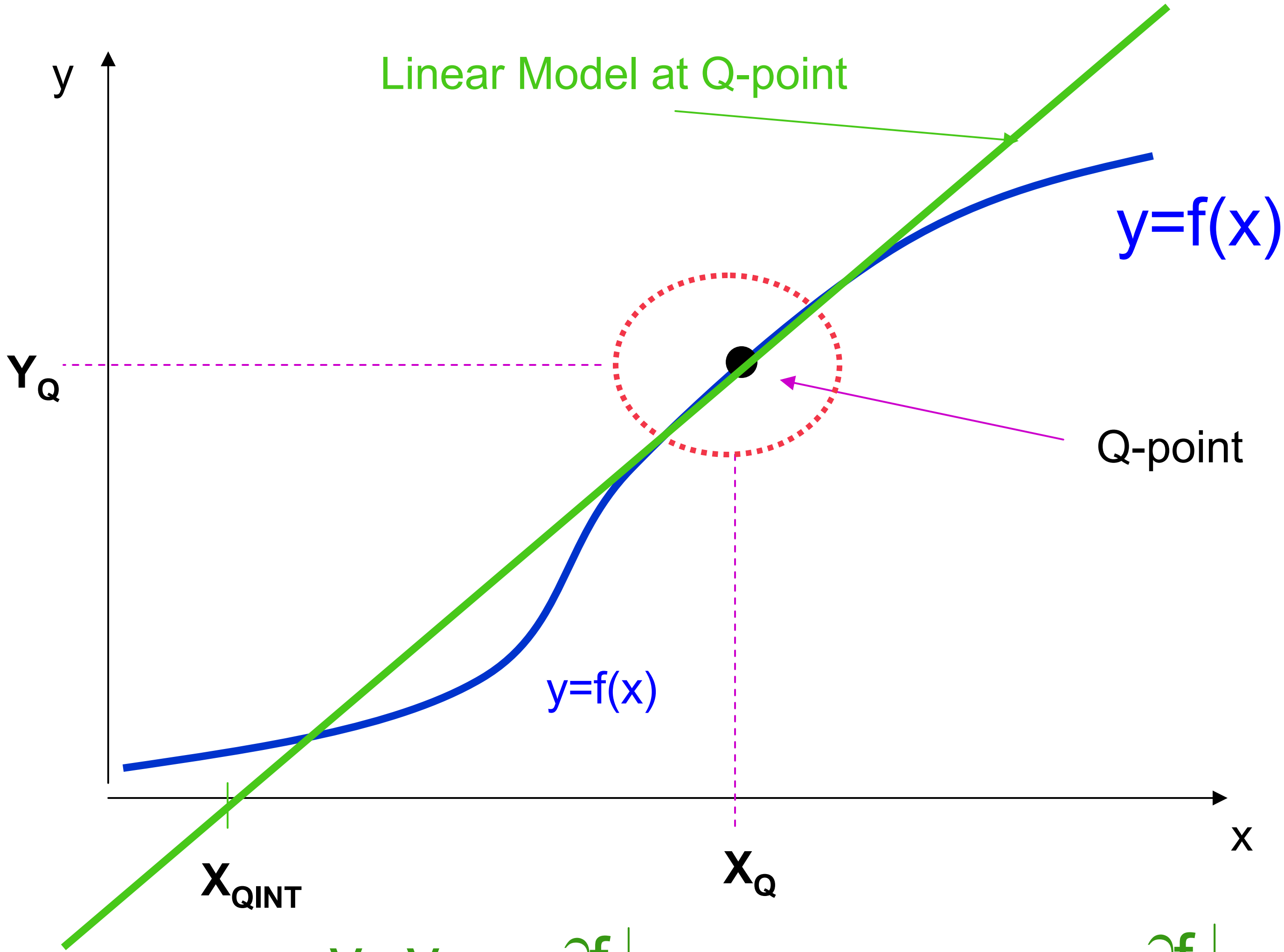
Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points

Small-Signal Principle



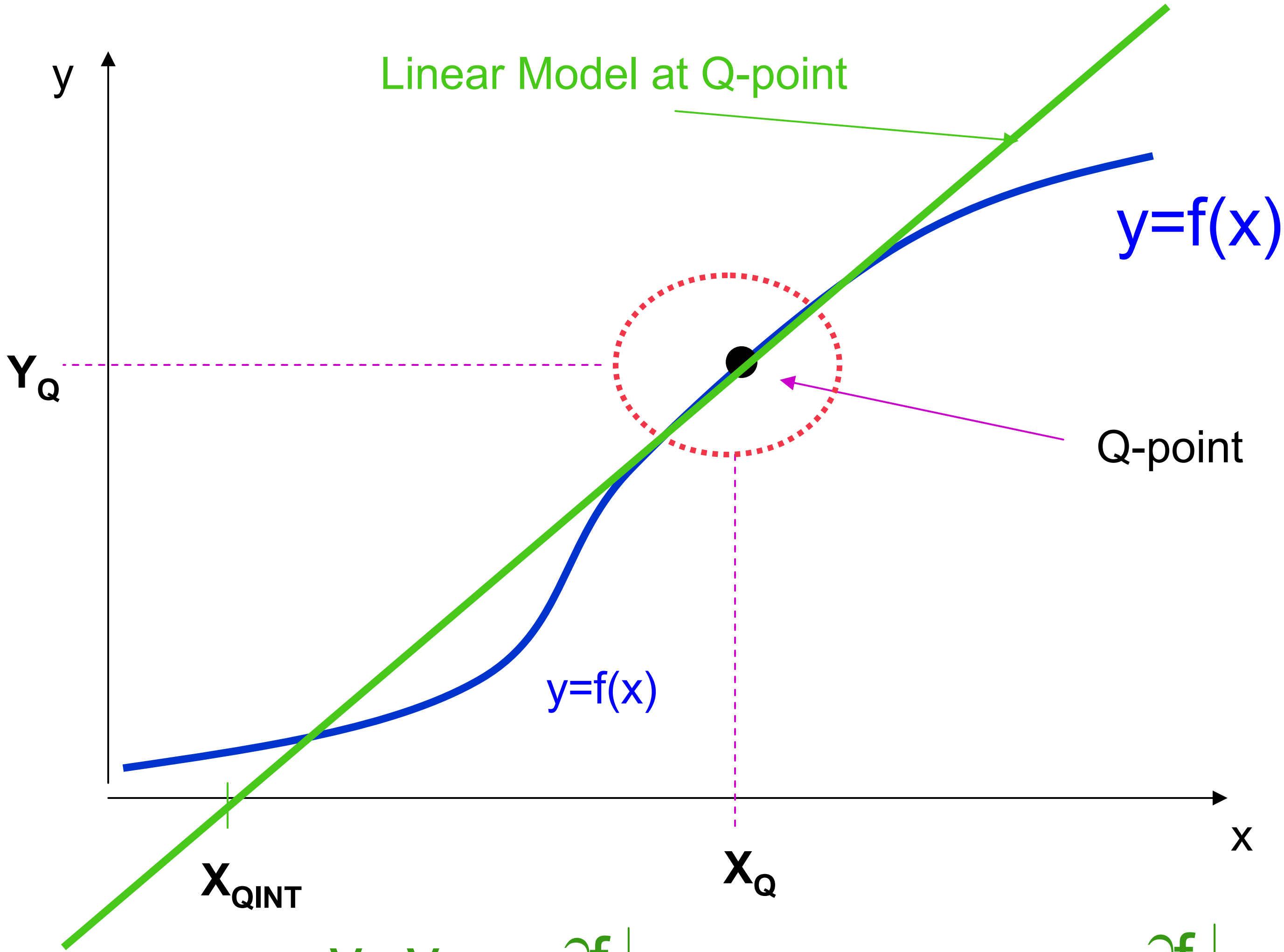
Device Behaves Linearly in Neighborhood of Q-Point
Can be characterized in terms of a small-signal coordinate system

Small-Signal Principle



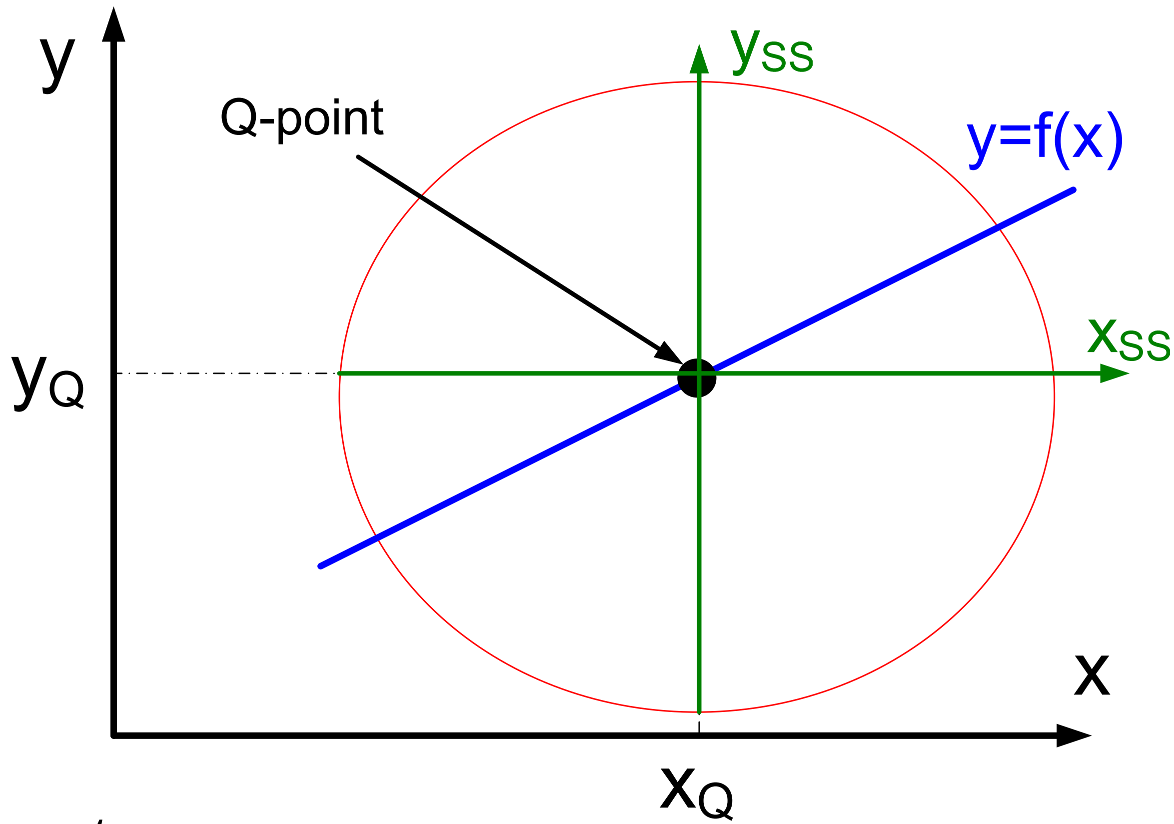
$$\frac{y-y_Q}{x-x_Q} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} \rightarrow y-y_Q = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q}(x-x_Q)$$

Small-Signal Principle



$$\frac{y - y_Q}{x - x_Q} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} \Longrightarrow y - y_Q = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q)$$

Small-Signal Principle



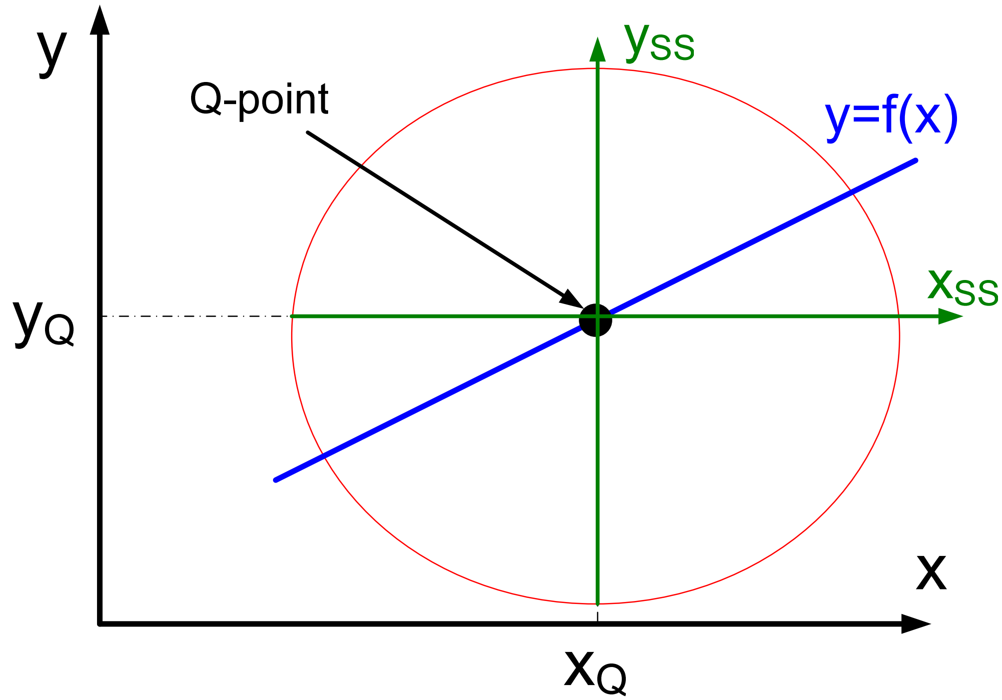
Changing coordinate systems:

$y_{SS}=y-y_Q$

$x_{SS}=x-x_Q$

$$y - y_Q = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q) \longrightarrow y_{SS} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{SS}$$

Small-Signal Principle

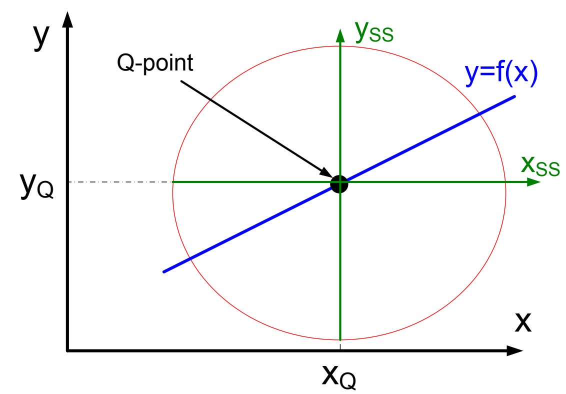


Small-Signal Model:

$$y_{ss} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{ss}$$

- *Linearized model for the nonlinear function y=f(x)*
- *Valid in the region of the Q-point*
- *Will show the small signal model is simply Taylor's series expansion at the Q-point truncated after first-order terms*

Small-Signal Principle



Observe:

$$y - y_Q = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q)$$

$$y = y_Q + \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q)$$

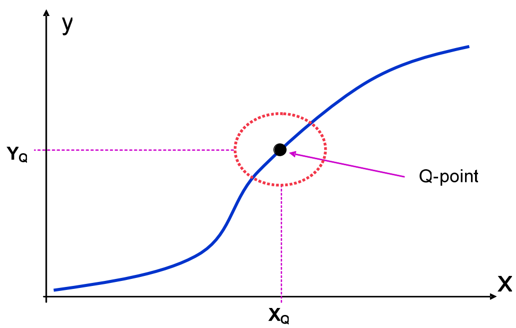
Small-Signal Model:

$$y = f(x_Q) + \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q) \quad \Longleftrightarrow \quad y_{SS} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{SS}$$

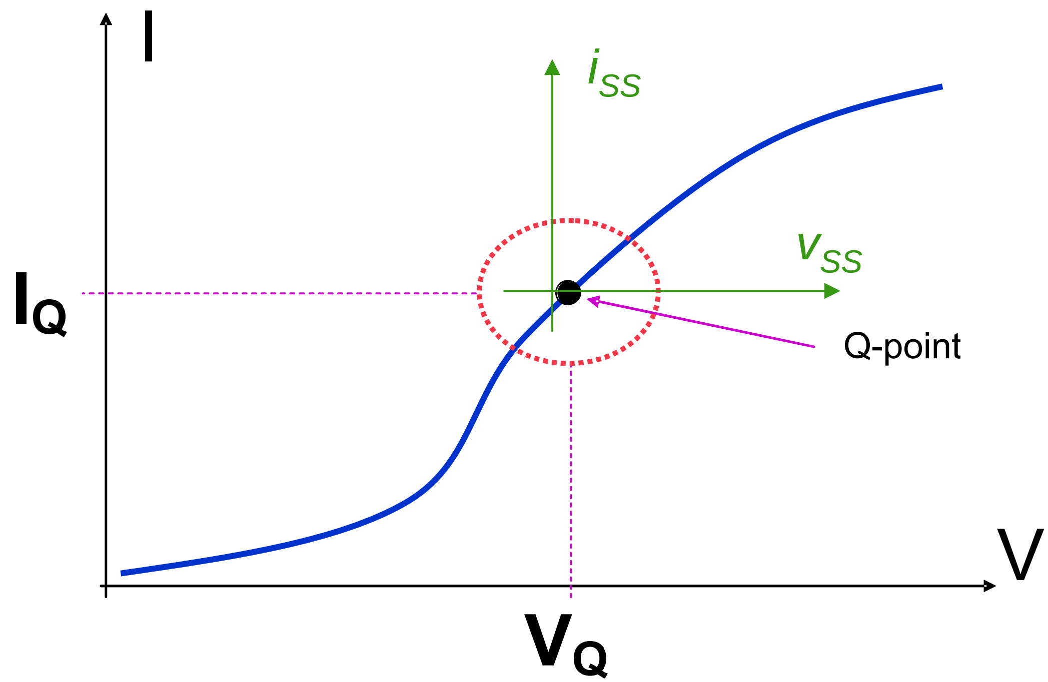
- *Mathematically, small signal model is simply Taylor's series expansion at the Q-point truncated after first-order terms*

Small-Signal Principle

(consider a 1-port circuit element)

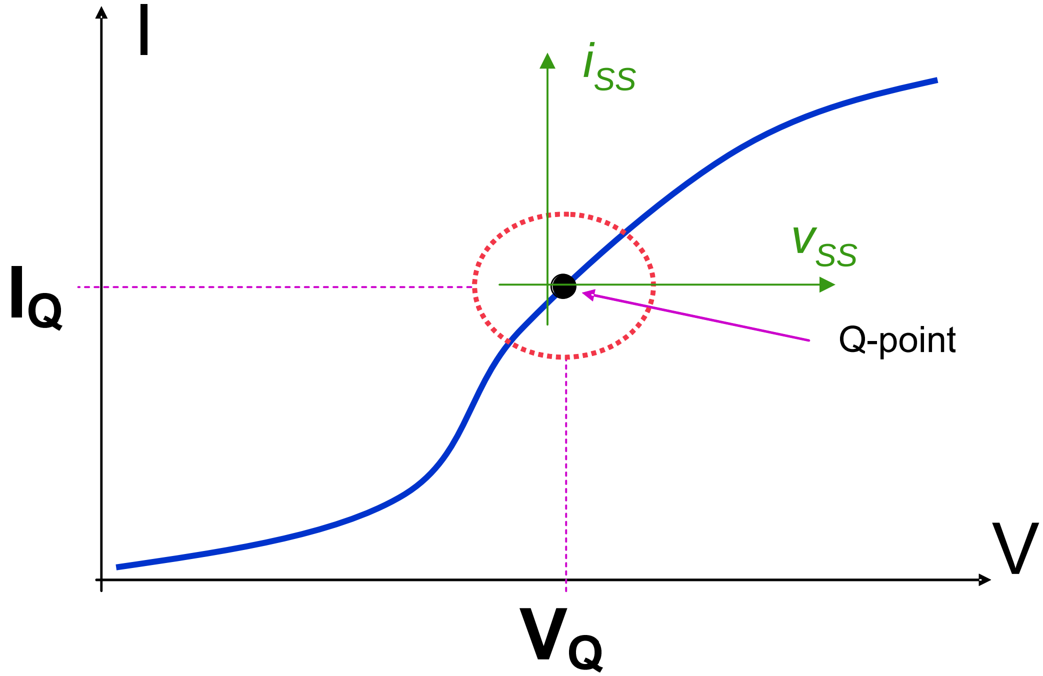


$$y_{ss} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{ss}$$



$$i_{ss} = \left.\frac{\partial I}{\partial V}\right|_{V=V_Q} v_{ss}$$

Small-Signal Principle



$$i_{ss} = \left.\frac{\partial I}{\partial V}\right|_{V=V_Q} v_{ss}$$

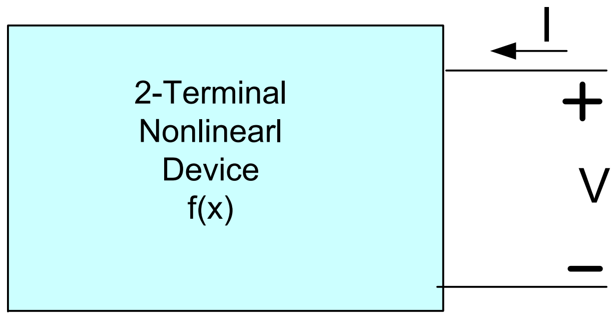
$$i_{ss} \overset{\text{def}}{=} \boldsymbol{i}$$

$$v_{ss} \overset{\text{def}}{=} \boldsymbol{v}$$

$$y \overset{defn}{=} \left.\frac{\partial I}{\partial V}\right|_{V=V_Q}$$

$$\boxed{\boldsymbol{i} = y\,\boldsymbol{v}}$$

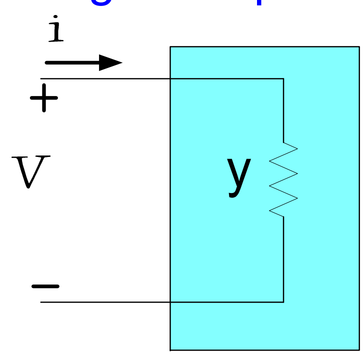
Small-Signal Principle



$$y = \left.\frac{\partial I}{\partial V}\right|_{V=V_Q}$$

$$\boldsymbol{i} = y\,\boldsymbol{v}$$

A Small Signal Equivalent Circuit



The small-signal model of this 2-terminal electrical network is a resistor of value 1/y
One small-signal parameter characterizes this one-port but it is dependent on Q-point

Small-Signal Principle

Goal with small signal model is to predict performance of circuit or device in the vicinity of an operating point

Operating point is often termed Q-point

Will be extended to functions of two and three variables

Small-signal Operation of Nonlinear Circuits

- Small-signal principles

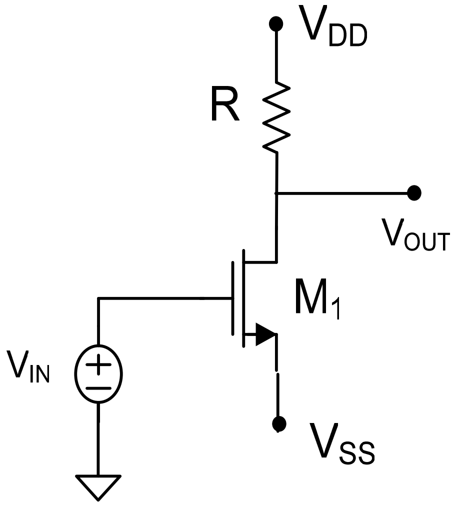
- Example Circuit

  (the analysis will be tedious and serve as justification for introducing more efficient methods of analyzing nonlinear circuits)

- Small-Signal Models

- Small-Signal Analysis of Nonlinear Circuits

Small signal analysis example



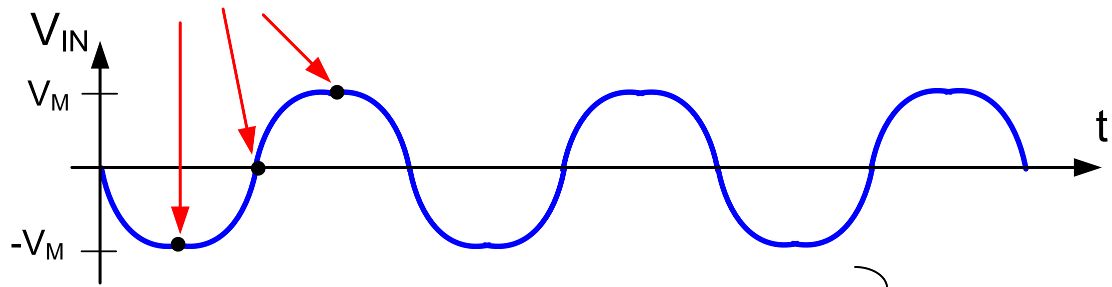
$V_{IN}=V_M \sin\omega t$

V_M is small

By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region

Consider three points on the input waveform

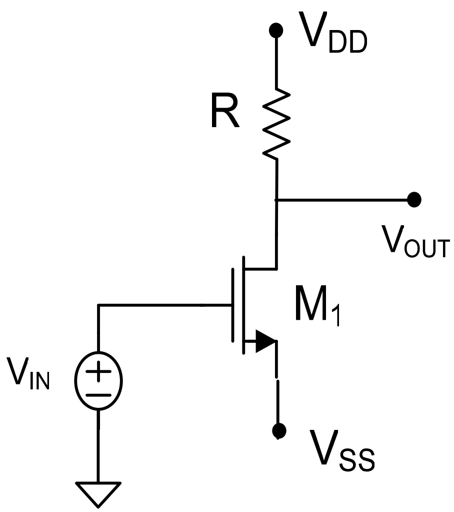


$$I_D = \frac{\mu C_{OX} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2$$

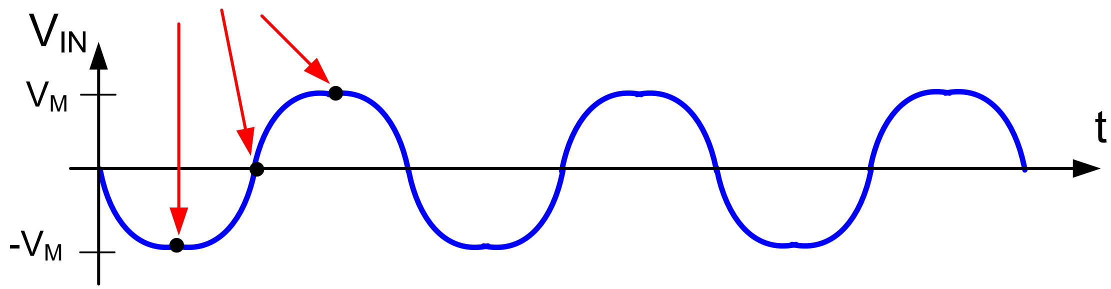
$$V_{OUT}=V_{DD}-I_D R$$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2 R$$

Small signal analysis example



Consider three points on the input waveform



$V_{IN}=V_M \sin\omega t$

V_M is small

at $V_{IN}=V_M$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_M - V_{SS} - V_T\right)^2 R$$

at $V_{IN}=0$

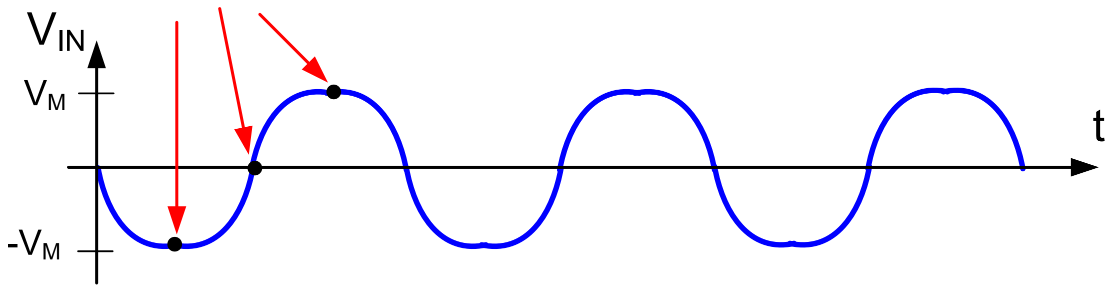
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(0 - V_{SS} - V_T\right)^2 R$$

at $V_{IN}= -V_M$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(-V_M - V_{SS} - V_T\right)^2 R$$

Small signal analysis example

Consider three points on the input waveform



at $V_{IN}=V_M$ $$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_M - V_{SS} - V_T\right)^2 R$$

at $V_{IN}=0$ $$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(0 - V_{SS} - V_T\right)^2 R$$

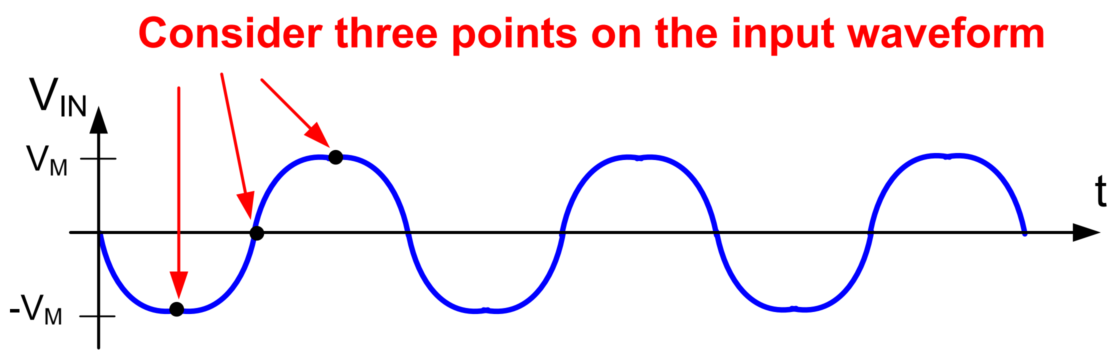
at $V_{IN}= -V_M$ $$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(-V_M - V_{SS} - V_T\right)^2 R$$

These are highly nonlinear equations !

Recall that is x is small $$\left(1+x\right)^2 \cong 1+2x$$

Observe this is a linearizing approximation to the nonlinear function $(1+x)^2$

Small signal analysis example



Consider the first of these 3 equations:

at $V_{IN}=V_M$
$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_M-V_{SS}-V_T\right)^2R$$

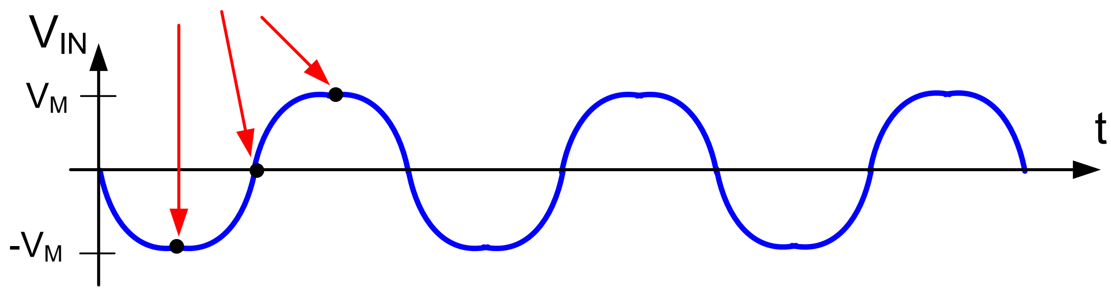
Recall that is x is small
$$\left(1+x\right)^2\cong 1+2x$$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_{SS}+V_T\right)^2\left(1-\frac{V_M}{V_{SS}+V_T}\right)^2R$$

$$V_{OUT}\simeq V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_{SS}+V_T\right)^2\left(1-\frac{V_M}{2\left(V_{SS}+V_T\right)}\right)R$$

$$V_{OUT}\simeq\left[V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_{SS}+V_T\right)^2R\right]+V_M\left[\frac{\mu C_{OX}WR\left(V_{SS}+V_T\right)}{4L}\right]$$

Small signal analysis example

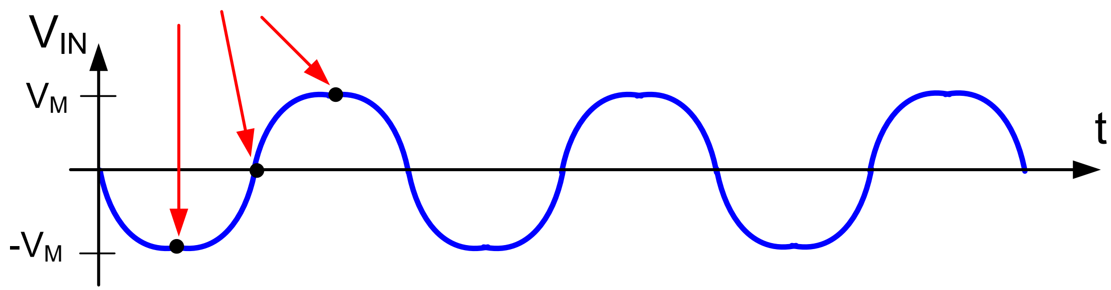


Consider the second of these 3 equations:

at $V_{IN}=0$
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(0 - V_{SS} - V_T\right)^2 R$$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{SS} + V_T\right)^2 R$$

Small signal analysis example



Consider the third of these 3 equations:

at $V_{IN}=-V_M$ $$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(-V_M - V_{SS} - V_T\right)^2 R$$

Recall that is x is small $$\left(1+x\right)^2 \cong 1+2x$$

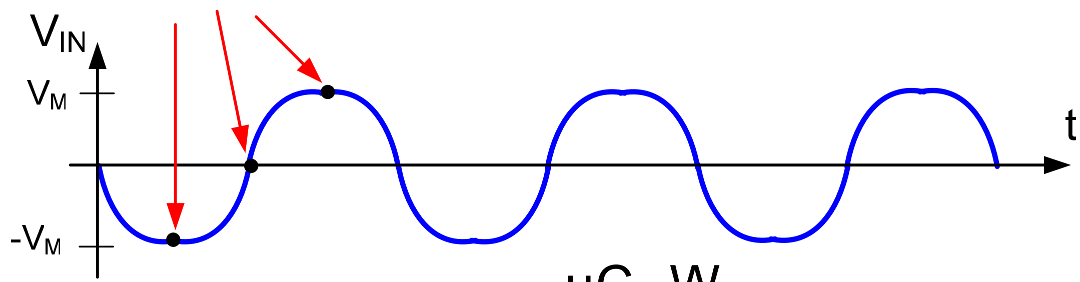
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2 \left(1+\frac{V_M}{V_{SS}+V_T}\right)^2 R$$

$$V_{OUT} \simeq V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2 \left(1+\frac{V_M}{2\left(V_{SS}+V_T\right)}\right) R$$

$$V_{OUT} \simeq \left[V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2 R\right] - V_M\left[\frac{\mu C_{OX} W R\left(V_{SS}+V_T\right)}{4L}\right]$$

Small signal analysis example

Consider three points on the input waveform



at $V_{IN}=V_M$ $$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_M-V_{SS}-V_T\right)^2R$$

at $V_{IN}=0$ $$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(0-V_{SS}-V_T\right)^2R$$

at $V_{IN}=-V_M$ $$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(-V_M-V_{SS}-V_T\right)^2R$$

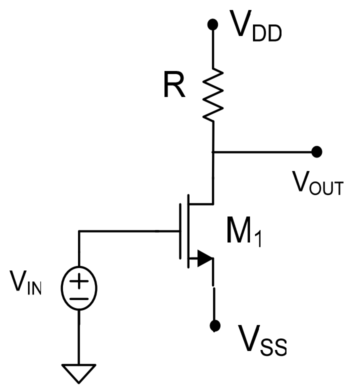
After all of this work, these 3 nonlinear equations "simplify to"

at $V_{IN}=V_M$ $$V_{OUT}\simeq\left[V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_{SS}+V_T\right)^2R\right]+V_M\left[\frac{\mu C_{OX}WR\left(V_{SS}+V_T\right)}{4L}\right]$$

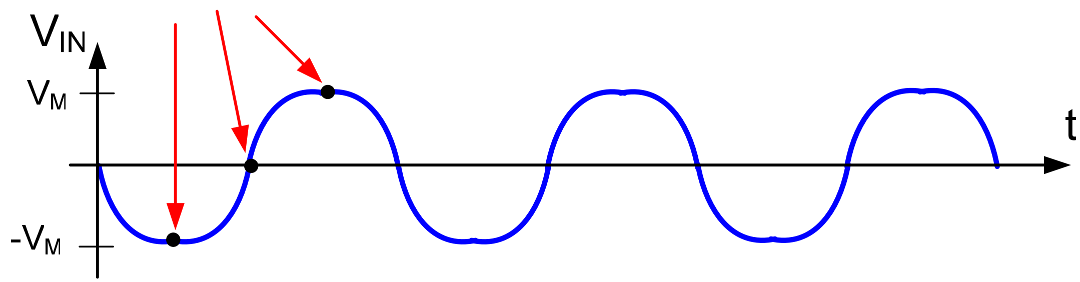
at $V_{IN}=0$ $$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_{SS}+V_T\right)^2R$$

at $V_{IN}=-V_M$ $$V_{OUT}\simeq\left[V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_{SS}+V_T\right)^2R\right]-V_M\left[\frac{\mu C_{OX}WR\left(V_{SS}+V_T\right)}{4L}\right]$$

Small signal analysis example



Consider three points on the input waveform



After all of this work, these 3 nonlinear equations "simplify to"

at $V_{IN}=V_M$
$$V_{OUT} \simeq \left[V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2 R\right] + V_M\left[\frac{\mu C_{OX} W R\left(V_{SS}+V_T\right)}{4L}\right]$$

at $V_{IN}=0$
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2 R$$

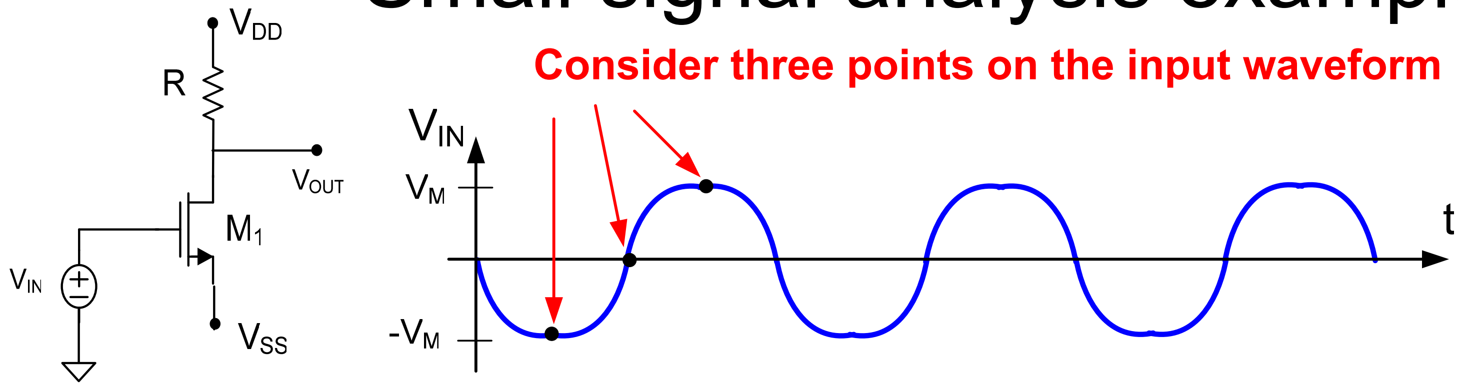
at $V_{IN}= -V_M$
$$V_{OUT} \simeq \left[V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2 R\right] - V_M\left[\frac{\mu C_{OX} W R\left(V_{SS}+V_T\right)}{4L}\right]$$

Note that the output deviation from the value when V_{IN}=0 have the same magnitudes but opposite signs and are linearly proportional to V_M

Note that the output decreases when the input increases and it increases when the input decreases

It can be shown that for some convenient values of W, L, and R, the coefficient multiplying V_M can be much larger than 1

Small signal analysis example



Note that the output deviation from the value when V_{IN}=0 have the same magnitudes but opposite signs and are linearly proportional to V_M

Note that the output decreases when the input increases and it increases when the input decreases

It can be shown that for some convenient values of W, L, and R, the coefficient multiplying V_M can be much larger than 1

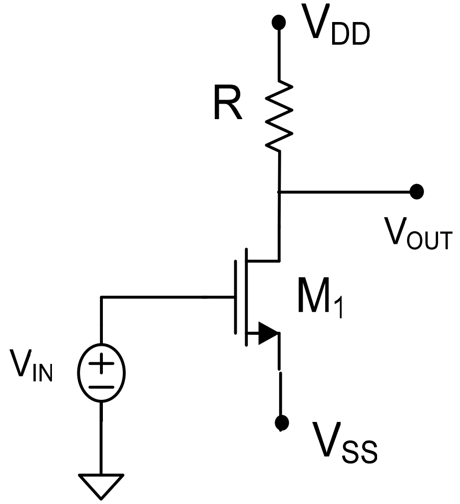
This simple nonlinear transistor circuit has gain !

Have the output values only for three values of the input

But this analysis method is TOO tedious !!!

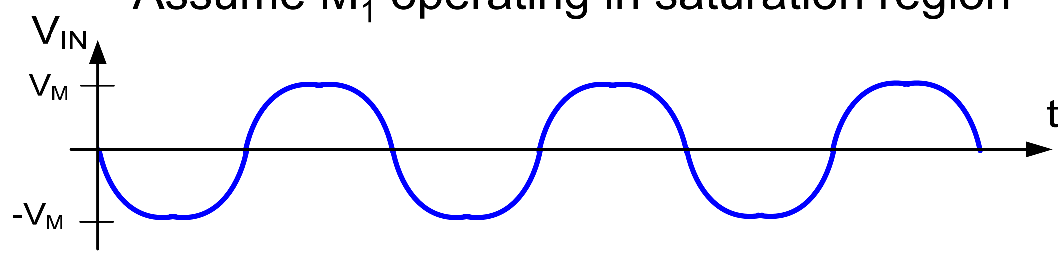
Small signal analysis example

Another way of analyzing this circuit



By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region



$V_{IN}=V_M \sin\omega t$

V_M is small

$$\left.\begin{aligned} V_{OUT} &= V_{DD} - I_D R \\ I_D &= \frac{\mu C_{OX} W}{2L}\left(V_{IN} - V_{SS} - V_T\right)^2 \end{aligned}\right\}$$

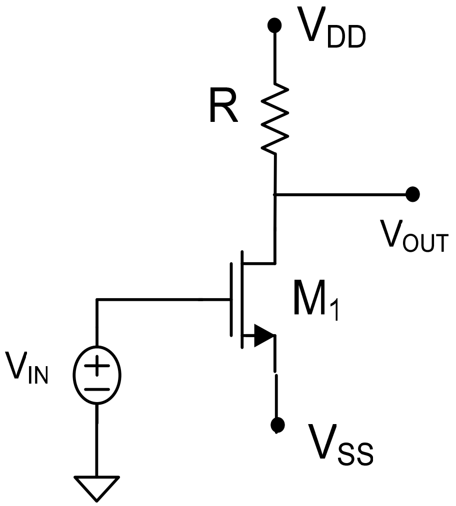
Define I_{DQ} to be the drain current when $V_{IN}=0$ and V_{OQ} to be the output voltage when $V_{IN}=0$ (will use this later)

$$I_{DQ} = \frac{\mu C_{OX} W}{2L}\left(V_{SS} + V_T\right)^2$$

$$V_{OUTQ} = V_{DD} - I_{DQ}R = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{SS} + V_T\right)^2 R$$

Small signal analysis example

Another way of analyzing this circuit

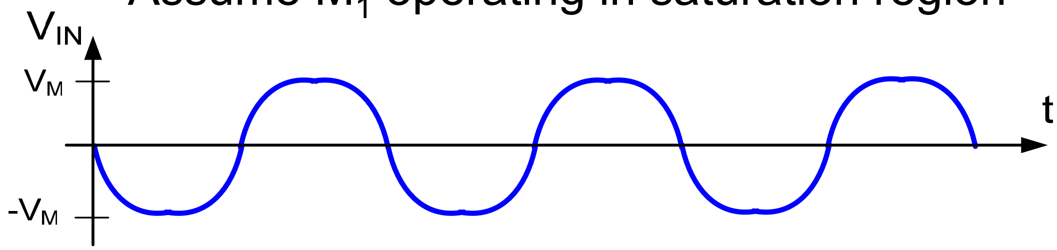


$V_{IN}=V_M\sin\omega t$

V_M is small

By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region



$$\left.\begin{aligned} V_{OUT} &= V_{DD} - I_D R \\ I_D &= \frac{\mu C_{OX} W}{2L}\left(V_{IN} - V_{SS} - V_T\right)^2 \end{aligned}\right\}$$

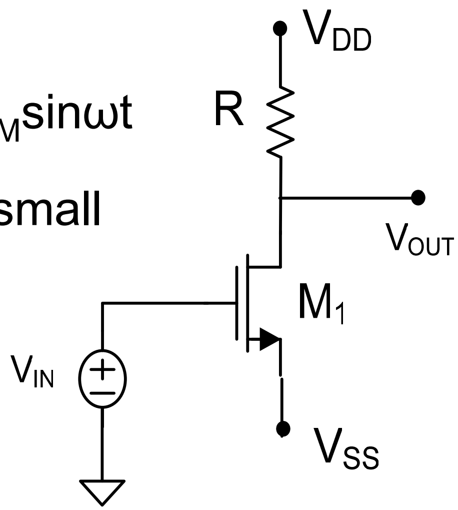
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_{IN} - V_{SS} - V_T\right)^2 R$$

$$\mathbf{V_{OUT}} = \mathbf{V_{DD}} - \frac{\boldsymbol{\mu} \mathbf{C_{OX} W}}{\mathbf{2L}}\left(\mathbf{V_M} \sin\omega \mathbf{t} - \left[\mathbf{V_{SS}} + \mathbf{V_T}\right]\right)^2 \mathbf{R}$$

Small signal analysis example

$V_{IN}=V_M \sin\omega t$

V_M is small



$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_M \sin\omega t - [V_{SS} + V_T]\right)^2 R$$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS} + V_T]^2\left(1 - \frac{V_M \sin\omega t}{[V_{SS} + V_T]}\right)^2 R$$

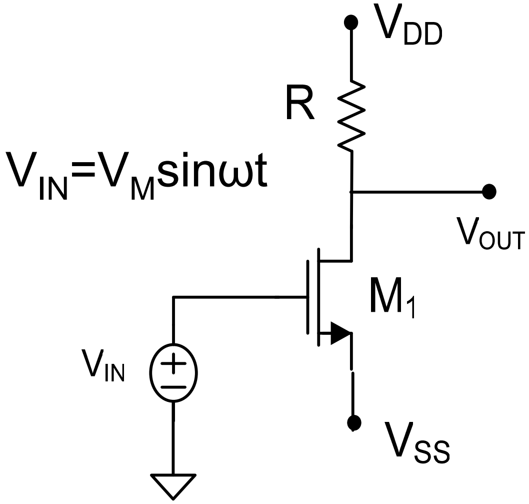
Recall that if x is small $(1+x)^2 \cong 1+2x$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS} + V_T]^2\left(1 - \frac{2V_M \sin\omega t}{[V_{SS} + V_T]}\right) R$$

$$V_{OUT} = \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS} + V_T]^2 R\right\} + \frac{\mu C_{OX} W}{2L}[V_{SS} + V_T]^2\left(\frac{2V_M \sin\omega t}{[V_{SS} + V_T]}\right) R$$

$$V_{OUT} = \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS} + V_T]^2 R\right\} + \left\{\frac{\mu C_{OX} W}{L}[V_{SS} + V_T] R\right\} V_M \sin\omega t$$

Small signal analysis example

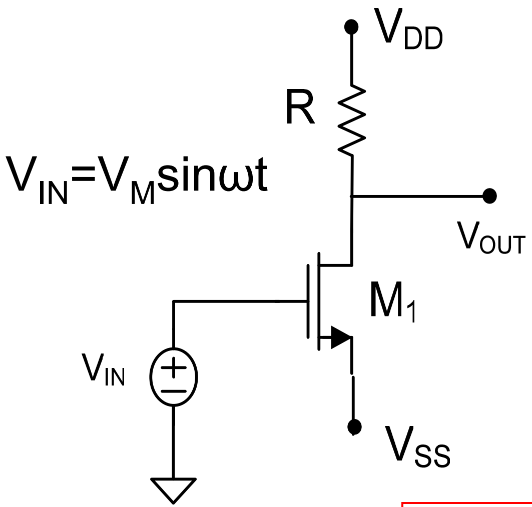


By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region

$$V_{OUT} = \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS} + V_T\right]^2 R\right\} + \left\{\frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R\right\} V_M \sin \omega t$$

Small signal analysis example



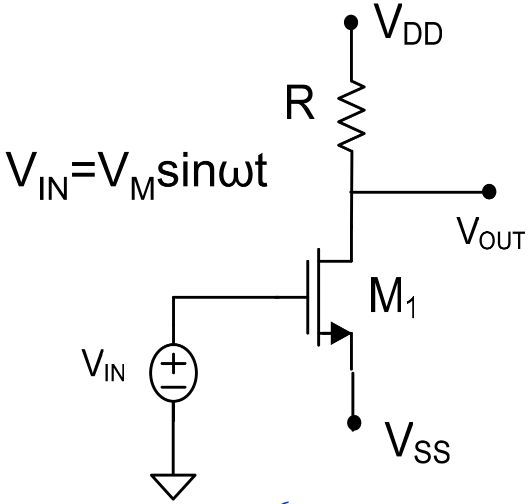
Assume M_1 operating in saturation region

$$V_{OUT} = \underbrace{\left\{ V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS} + V_T\right]^2 R \right\}}_{\textit{Quiescent Output}} + \underbrace{\left\{ \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R \right\}}_{\textit{ss Voltage Gain}} V_M \sin\omega t$$

$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$

But – this expression gives little insight into how large the gain is !

Small signal analysis example



$$V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS}+V_T\right]^2 R \right\} + \left\{ \frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right] R \right\} V_M \sin\omega t$$

$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right] R$$

But recall:

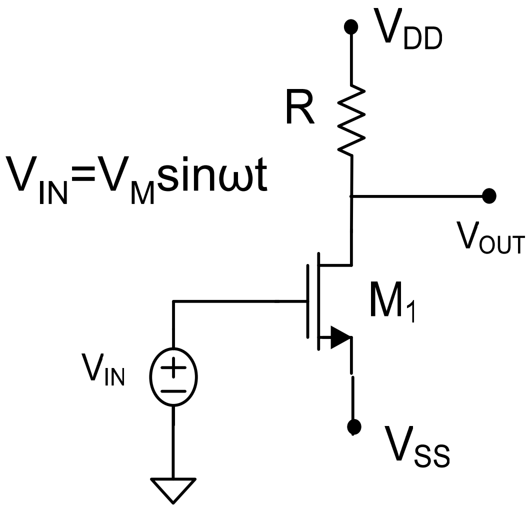
$$I_{DQ} = \frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2$$

Thus, substituting from the expression for I_{DQ} we obtain

$$A_v = \frac{2 I_{DQ} R}{\left[V_{SS}+V_T\right]}$$

Note this is negative since $V_{SS}+V_T < 0$

Small signal analysis example



$$A_v = \frac{2I_{DQ}R}{[V_{SS}+V_T]}$$

Observe the small signal voltage gain is twice the Quiescent voltage across R divided by $V_{SS}+V_T$

If V_{SS} and R are chosen properly, this inverting gain can be quite large!

- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements

End of Lecture 32